

Mathematics and climate change: What role do you think mathematics can play in guiding policy makers and in helping public understanding?

Abstract

Danish physicist Niels Bohr said, “it is very difficult to predict — especially the future.” In order to attempt to make predictions, the approach of a mathematician must encompass both fluent knowledge of the discipline and ability to apply this to the real-world. By analysing variance and calculating averages, mathematicians can discover whether the climate has changed and how. Presenting mathematical evidence can permit more informed and accurate decision-making by policy makers. Consequently, this encourages the IPCC (International Panel on Climate Change) and the public to work together towards preventing the potentially devastating social, economic and environmental effects of climate change. Due to the effects of greenhouse gases, earth’s atmosphere retains a greater amount of solar energy emitted by the sun. We will explore how mathematics demonstrates to the public the environmental impact that this has on sea-ice. This essay also investigates mathematical models of earth’s changing climate, and conveys the synergy between applied and pure methods of mathematics and evidence-based policy making.

Exploring the role of Mathematics

Mathematics evolves in parallel to the needs of societies. The beginnings of mathematics permitted basic functions, such as trading local goods and counting time. To this present day, the discipline has advanced to allow us to gain a better understanding of more complex systems and processes. We can now use mathematics to systematically analyse and quantify climate change and its global impacts. We will explore mathematical models which directly isolate the important physical processes that determine Earth’s climate. The results of these do not only lead to a justified view on the topic of climate change, but also show the relevance of mathematics in civilisation.

Evidencing Climate Change

‘Does climate change really exist?’: this question stands centrally. The IPCC defines climate change as ‘*a change in the state of the climate that can be identified (e.g., by using statistical tests) by changes in the mean and/or the variability of its properties*’ [1]. Professor Chris

Budd OBE has considered these varying climate properties to be ‘*at least five indicators that make us think climate change is occurring*’ [2]. One of these documented indicators is the loss of Arctic Sea ice measured and recorded over 35 years. Examining the data in *Figure 1*, Budd deemed the recent dramatic decrease in Arctic Ice to be an ‘*undisputable fact*’ *ibid* [2]. Due to its resultant impact on earth’s albedo value, later in this essay we will further explore the significance of Arctic Sea ice loss in another calculation.

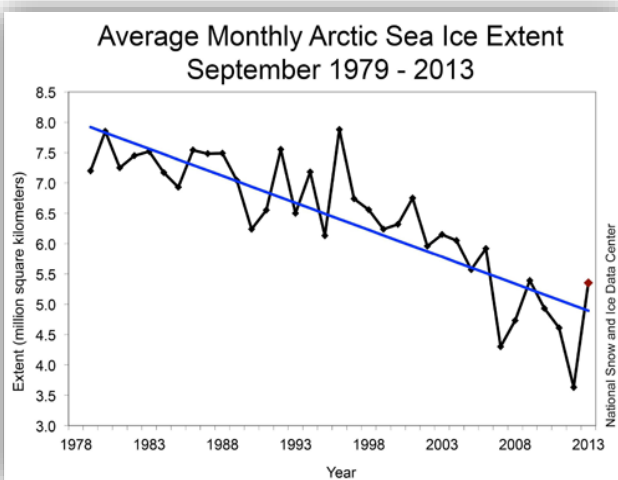


Figure 1

The statistical method ‘linear regression’ is used here to create a regression line with an equation in the form of $Y = mX + b$. Here we assume that Year (X) and Extent (Y) are related to each other by the line function $Y = mX +$

b , for some numbers m (slope) and b (the y-intercept). Using simple linear regression, we can model the relationship between these two continuous variables. This allows us to create a predictive climate model using one independent/predictor variable X and one dependent/outcome Y variable. Inferring from the straight line fitted in *Figure 1*, the prediction is that all Arctic Ice will vanish by the end of the century. This will cause a dramatic increase in sea-level.

However, the study of earth's changing climate remains highly complex. The appearance of random statistical variation continues to enhance the difficulty of making projections for the long-term. In order for mathematicians to present informed evidence to policy makers, such as the IPCC and members of parliament, all levels of uncertainty in statistical predictions must be scrutinised.

Furthermore, a carefully considered mathematical model can provide an accurate visual representation of an abstract concept. We could benefit further from using simple regression models for optimisation: policies, which align with an 'optimisation goal' to hit a target in an acceptable amount of time, can be formulated by identifying factors that lead to a maximum or minimum change in the climate. Current policies that are examples of optimisation goals are the UK's commitment to 'reduce emissions by 78% by 2035 compared to 1990 levels' [3] and 'decarbonise all sectors of the UK economy to meet our net zero target by 2050' [4]. Linking the general implications of climate change to the laws and rules of mathematics is critical in aiding public understanding, so that climate change can be evidenced and understood.

An Energy Balance Model

Moreover, we can explore the role of another simplified mathematical model and its objectives. An Energy Balance Model aims to '*predict the average surface temperature of the Earth from solar radiation, emission of radiation to outer space, and Earth's energy absorption and greenhouse effects*' [5]. Despite the complexity of climate change, we continue to benefit from simple models which are easier to understand and facilitate analysis and numerical study of how sensitive our climate is. In a 2016 article, Professor Chris Budd OBE outlined the stages below of an Energy Balance Model [6].

As all models have associated assumptions, we must assume that the radiation emitted from the Sun heats the Earth, and that this radiation has an average temperature T . We reach energy equilibrium when both heat energy emitted from the sun and heat energy reflected back into space balance (hence Energy Balance).

Heat energy emitted from the Sun is given by:

$$(1 - a)S, \quad (1.1)$$

Where S is the incoming power from the Sun (approximated in *Budd's* article to $342Wm^{-2}$ on average). a is the *albedo* value of earth: a measure of how much energy is reflected back into space. Currently $a = 0.31$

The heat energy radiated back into space is given by:

$$\sigma eT^4 \quad (1.2)$$

Here

$$\sigma = 5.67 \times 10^{-8}Wm^{-2}K^{-4} \quad (1.3)$$

Greek letter sigma σ is the Stefan-Boltzmann constant (temperature measured in Kelvin K).

e is the *emissivity* (measure of how transparent, less 'thick', our atmosphere is). '*The atmosphere acts somewhat like a blanket that becomes thicker when amounts of greenhouse*

gases increase [7]: as a result, *emissivity* decreases as greenhouse gas output from Earth increases. Currently on Earth we have $e = 0.605$. Whereas on the Moon almost no atmosphere has *emissivity* of $e = 1$

Considering both heat radiated back into space and energy emitted from the sun to be at equilibrium, we balance expressions (1.1) and (1.2):

$$\sigma e T^4 = (1 - a)S,$$

Rearranging for T:

$$T = \left(\frac{(1-a)S}{\sigma e} \right)^{1/4} \quad (1.4)$$

Solving using the above values for σ , e , a , and S to find Earth's T:

$$T = \left(\frac{(1-0.31)(342)}{(5.67 \times 10^{-8})(0.605)} \right)^{1/4} = 287.994498K = 288K \text{ (3SF)}$$

288 Kelvin corresponds to 15°C, which is the exact value of Earth's temperature measured by NASA [8].

The power of this expression is evident when we consider different a (albedo) values. As documented previously, we can evidence climate change by considering loss of Arctic Sea ice. Melting of this ice results in less white, reflective cover of earth, so a decreases meaning that $(1 - a)$ and hence temperature T increases. Additionally, if *emissivity* e decreases then temperature T , once again, increases. To conclude, we now see the causal link between increase in the greenhouse effect in our atmosphere and the rising predicted mean temperature of earth.

Mathematicians break down complex problems, systematically and logically, into solutions of clarity. Commitment to reducing anthropogenic (man-made) factors, which increase earth's temperature and resultantly melt Arctic Sea ice, is the solution here. The simple mathematical evidence supporting this solution is more comprehensible to non-specialists. This evidence also outlines the synergy between mathematical modelling and predicting the future of our changing climate. An article entitled 'teaching climate change in mathematics classrooms: an ethical responsibility', encourages questioning '*how incorporating issues of climate change into the teaching and learning of mathematics can be understood as a moral and ethical act?*' [9]. Perhaps this question highlights the responsibility of learners to utilise mathematics to ensure climate rights. Particularly as the energy balance model above does not require mathematics skills beyond A-level classrooms. Results of this equation and those similar could encourage tangible, deliverable and evidence-based policy options to be implemented by those in power.

Gerald North's Ice Cap Model

In continuation, a warming climate can cause seawater to expand and ice over land to melt. Both of these can cause a rise in sea level. Featured in the 'Special Report: global warming of 1.5 degrees' from the IPCC, researchers expect '*the oceans to rise between 10 and 30 inches (26 to 77 centimetres) by 2100, with temperatures likely to warm to 1.5 °C between 2030 and 2052.*' [10]

Gerald North studied a model for average sea-level temperature T as a function of $x = \sin(\text{latitude})$ and time t . Latitude is **the measurement of distance north or south of the Equator**. The model considers warming from '*solar radiation at different latitudes,*

diffusion of heat from warm areas to cold areas, and the albedo value of ice cap and ice-free regions' [11].

The rise over time of sea-level temperature is modelled as a partial differential equation here:

$$\frac{\partial T}{\partial t} = D \frac{\partial}{\partial x} \left[(1 - x^2) \frac{\partial T}{\partial x} \right] - A - BT - QS(x)a(T) \quad (2.1)$$

In this formula, Q is the solar constant (average amount of solar energy received by a square meter of the earth's upper atmosphere at the equator) divided by 4. We divide by 4 since the solar energy is spread over the surface of the spherical planet. Q is measured in Wm^{-2} .

D , A and B are empirical constants. $S(x)$ is the mean annual sunlight distribution.

$$a(T) = 0.38 \text{ for } T < -10^\circ\text{C}, 0.68 \text{ for } T > -10^\circ\text{C}$$

$a(T)$ represents the co-albedo.

The methods required to solve this partial differential equation go beyond the scope of this essay. However, we can draw our attention to a simpler model for the size of ice caps (and how ice caps evolve over time).

Gerald North derived this ordinary differential equation from (1.5) *Chapter 2 ibid* [11]:

$$-D \frac{d}{dx} \left[(1 - x^2) \frac{dT}{dx} \right] + A + BT = QS(x)a(T) \quad (2.2)$$

Where (all parameters below taken from [11]):

x $\sin(\text{latitude})$ T $\text{sea level temperature}$

A, B $\text{empirical radiation coefficients } (201.4 Wm^{-2}, 1.45 Wm^{-2}C^{-1})$

D $\text{thermal diffusion coefficient } \left(\sqrt{\frac{D}{B}} = 0.30 \right) \text{ therefore } D = 0.1305$

$a(T)$ $\text{co-albedo} = 0.38 \text{ for } T < -10^\circ\text{C}, 0.68 \text{ for } T > -10^\circ\text{C}$

Q $\frac{\text{solar constant}}{4}$

$S(x)$ $\text{mean annual sunlight distribution} = 1 - 0.482P(x); P(x) = (3x^2 - 1)(0.5)$

To achieve symmetric solutions, solutions to the ordinary differential equation for the

Northern Hemisphere must satisfy appropriate boundary conditions:

$$\frac{dT}{dx} = 0 \text{ at } x = 0 \text{ and } x = 1$$

These boundaries are potential steady-state temperature distributions. For each such temperature distribution, the lowest value of x above which $T(x) < -10^\circ$ is an equilibrium ice cap size.

Equation (2.2) can be solved by a variety of techniques. A typical form of displaying the solution is to plot x_s , **the sine of the latitude of the ice edge, as a function of solar constant Q .**

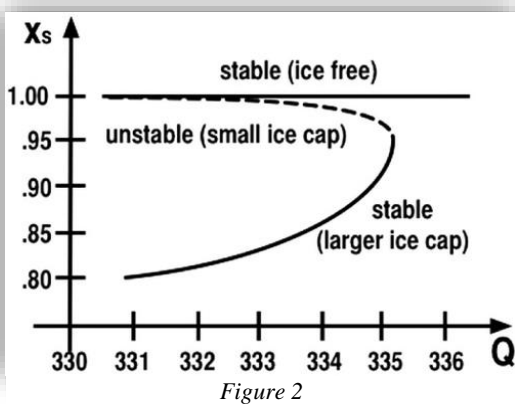


Figure 2

Figure 2 shows a portion of the equilibrium curve for north-south symmetric solutions adapted from Drazin and Griffel (1977). The segments of the curve with negative gradient (dashed line) represent unstable ice caps (Cahalan and North, 1979). Examining this figure,

values of the ice edge x_s greater than about 0.955 are unstable. Hence, for the values Q in the range shown on the graph, only ice caps extending more than 73° towards the equator are stable (as $\sin(73) = 0.956$ *SF*). This highlights the meaning of small ice cap instability. Considering global warming, we can view an increase in the value of Q as representative of an increase in greenhouse gas levels, as such gases increase the absorption of solar energy in earth's atmosphere.

In continuation, we can also observe ice caps of the same size. Previously, we've viewed Q as constant, but the value of Q may vary with a shape-change of earth's orbit or with changes in the solar power emitted by the sun. North's analysis leads to the surprising conclusion in Figure 2: that the equilibrium of ice cap size is not solely determined by the solar constant Q . This is understood as for a certain range of values of Q (x axis) in the model, researchers stated three different steady state ice cap sizes [12]:

- The range of Q values where there are three ice caps is the middle equilibrium. One is unstable and the other two are stable.
- Of particular significance is when $Q = 332$, there are three steady state x_s values. One is $x_s = 1$, meaning that there is no ice cap all the way up to latitude 90° . This ice cap is stable.
- The second x_s equilibrium for this value of Q is slightly smaller, around 0.99 (meaning a small, but nonempty ice cap), but this is unstable. Instability of this ice cap means that if the ice cap is a slightly smaller, it shrinks to nothing, and if it is larger, it grows indefinitely.
- Finally, there is a third stable equilibrium $x_s \approx 0.8$, at latitude $\approx 55^\circ$.
- As Q increases towards 335, the two lower x_s equilibrium values get closer, and they ultimately meet and vanish around $Q = 335$.



Figure 3

Some scientists believe that the **West Antarctic ice sheet** is already becoming unstable, so it's among the regions that are most vulnerable to future ice sheet collapse. This area may also contain enough ice to cause several feet of sea-level rise. There are several consequences to unstable ice caps collapsing: loss of a rare ecosystem, and faster movement of glaciers could lead to further sea level rise. In climate research, tipping points have become a key concept. The melting of ice caps is considered a tipping point – as this would result in abrupt changes to earth's climate.

Nevertheless, some may consider that an ice-free Arctic would provide new benefits and opportunity for human invention. In summer months Arctic would become accessible for fishing and searching for oil. However, of high relevance to policy makers is the ruinous impact that increasing sea level and sea temperature could have on socio-economic systems. In terms of sectors, energy use and agriculture could be highly exposed. We cannot prioritise profiting from lucrative commodities, such as livestock and oil, over saving the climate of earth. Here, mathematical modelling of sea-ice behaviour provides a much-needed glimpse into the future.

Conclusion

Mathematics stands centrally in aiding public understanding and directing policy makers. The synergy between applied and pure methods of mathematics and evidence-based policy making is clear. Predictive climate models demand concise strategies over hopeful goals.

Seen in results of energy balance and regression models, abstract methods in the discipline can promote personal feelings of worry and a sense of urgency with regards to earth's climate. We know that the catalysts for climate change are human activities which pollute the environment. What will be the catalyst for a change in human behaviour to revitalise Earth? Perhaps it will be the help of mathematical models to format tangible, deliverable and statistically verified policy options. Such models will allow us to incorporate the effects of human activities and help us identify climate 'tipping points' at a global level. These will further public understanding and evade scepticism. Tackling climate change remains a collective venture. It is the collaboration of mathematics with other disciplines, such as politics, environmental science and sustainable engineering, which will ensure environmental stability. This is secured by appropriate policies and reinforced by public action.

Word Count [2497]

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