Will artificial intelligence one day make human mathematicians obsolete?

Abstract

Over the years, mathematicians have welcomed, with caution, the use of artificial intelligence (AI) and computers to help solve mathematical problems and proofs. The use of AI has helped expand the mathematical field more than anyone could predict, and the future holds the same exciting prospect. In 1970, the famous mathematician Paul Cohen, voiced his opinion that mathematicians would become obsolete due to the advances in technology [1]. However, at around the same time and in contrast to this, another leading mathematician, Paul Halmos, believed that AI and computers 'had no role in pure mathematics' [2]. In this debate, one of the key questions is whether the writing of proofs could become automated. This essay will explore the history of the use of AI in mathematics, the advances made in the field of proofs thanks to AI, and debate whether AI will ultimately make mathematicians obsolete.

What is Artificial Intelligence (AI)?

AI is the ability of a digital computer to perform tasks commonly associated with intelligent beings [3]. As such AI has the potential to reason, learn from its own experiences, develop and discover patterns and trends. AI has a wide range of applications, across many academic and industrial sectors. Examples include banking (providing customer support), finance (determining future patterns in the market), agriculture (helping farmers use resources more sustainably), healthcare (enabling innovation in diagnostics) and many more sectors [4].

AI is split in two broad categories:

1. Narrow AI, or "weak AI" - used for performing specific tasks, like playing chess or Go, translations, or for autonomous cars; and
2. Artificial General Intelligence (AGI), or "strong AI" - which has the capability to accomplish almost any tasks and has the intelligence of a human.

What do Mathematicians do?

Mathematics is the oldest and most fundamental science. From the beginning of civilisation, man used mathematics to trade goods and count seasons. The mathematics used was purely based on empirical experiences. The notion of a proof was introduced by Euclid, and 'by the use of deductive logic and self-evident truths, axioms' the first theorem was yielded, and the notion of the proof was introduced. This change from the 'empirical to deductive derived truths, is one we still practice today' [5]. Nowadays, mathematicians use mathematical theory, algorithms, computational techniques and the latest computer technology to solve economic, scientific and business problems.

A look at Proofs

'A proof is a step-by-step logical argument that verifies the truth of a conjecture' [1]. A mathematician begins with an axiom (a self-evident truth), and then uses deductive logic to construct a proof. A very simple example of deductive logic [5], is:

'Plants that are green have chlorophyll.
Tomato plant is green.
Conclusion: therefore, tomato plant has chlorophyll.'

Through this example, we can conclude that proofs are valid statements backed up with logical, well-defined arguments by use of axioms. All proofs in mathematics must abide to three general characteristics [5]; they must be:

1. Surveyable - the proof must be logical enough to be peer reviewed by a mathematician;
2. Formalizable - the proof must follow from axioms and the conclusion must be deducted from these by deductive reasoning; and
3. Convincing - mathematicians in the relevant field must be convinced of its correctness.

The first step of a proof requires inductive reasoning and intuition to form a conjecture. Intuition is holistic, creative, original, and it is essential in mathematics. Proofs require organised, methodical, logical and laborious methods of working; however, they also require creative thinking. Some of the best proofs ever to be solved required 'brave leaps of imagination based on intuition'. The field of proof relies on 'a mathematician's intuition to fill in the blanks' [2].

An example of a simple proof (by induction) that requires inductive reasoning is the formula for the sum of the elements in a row of the Pascal's triangle. Another interesting proof (by contradiction), is that the square root of 2 is irrational. Both proofs are explained below.

1 - Observation

Row	Sum
1	1
1 1	2
1 2 1	4
1 3 3 1	8
1 4 6 4 1	16

2 - Conjecture: The sum of the elements in the n^{th} row of Pascal's triangle is 2^n

3 – Proof by Induction

The sum $s(n+1)=2s(n)$ for any positive integer n

If $s(n)=2^n$ then $s(n+1)=2\times2^n=2^{n+1}$

Thus, the sum of elements in the n^{th} row of Pascal's triangle is $\mathbf{2^n}$

Proof by induction

1 - If $\sqrt{2}$ is rational, I can write $\sqrt{2} = \frac{a}{b}$,
with a and b whole numbers, b not zero,
and where either **a or b must be odd**

2 - It follows that $a^2 = 2 \cdot b^2$,
so a^2 is even, so a must also be even

3 - So we can write $a = 2k \Rightarrow 2 = \frac{4k^2}{b^2} \Rightarrow b^2 = 2k^2$

4 - This means that b^2 is even, so **b must also be even**,
which is a **contradiction** with the initial assumption!

Proof by contradiction

What is a Computer Assisted Proof?

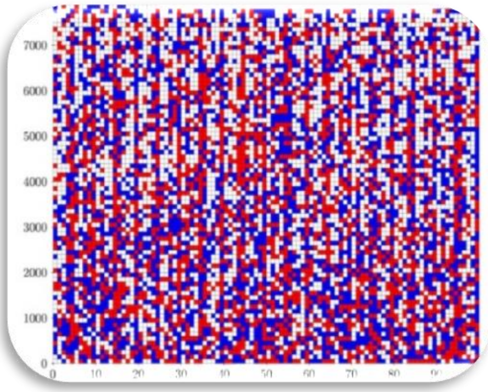
For decades, mathematicians have used computer programs as proof assistants to help them write proofs. Computer theorem provers can be split into two disciplines [1]: Automated Theorem Provers (ATP's), which use 'brute force methods to crunch through big calculations'; and Interactive Theorem Provers (ITP's), which act as 'proof assistants that can verify the accuracy of an argument and check existing proofs for errors'. Theorem provers have been used to help solve heavy calculation proofs that would otherwise have taken longer than a mathematician's lifetime to solve.

As mentioned before, conjectures require inductive reasoning and intuition, whereas proofs require intuition as well as the laborious tasks. However, in our current technological landscape, artificial intelligence is unable to do both. Even by merging ATP and ITP, automated reasoning will not be produced. So, can AI make mathematicians obsolete? Well, as discussed above, the most successful proofs require creativity and intuition. Computers have the ability to crunch, analyse, compare and recognise millions of proofs (more than any human could); however, their lack of intuition and creativity is their current limitation in automated reasoning. The main role for AI is to carry out calculations that are mundane and laborious. However, an interesting question is, even if automated reasoning is developed, 'could computers ever generate interesting conjectures?' [2]. The use of AI in formal proofs has undoubtedly been immeasurably useful, as they compute the intermediate steps where there is no need for intuition. How long will it be until AI does more than just the mundane steps? Some argue that creativity is a human quality that is impossible to automate, however, if AI develops intuition, can it generate a conjecture with a proof that is easily understandable? Could they conceivably make mathematicians obsolete?

AI has already been used to solve proofs that could never have been done by humans. This essay will explore three significant examples: the Boolean Pythagorean Triples, the Four-Colour Theorem, and the Kepler Conjecture.

Boolean Pythagorean Triples

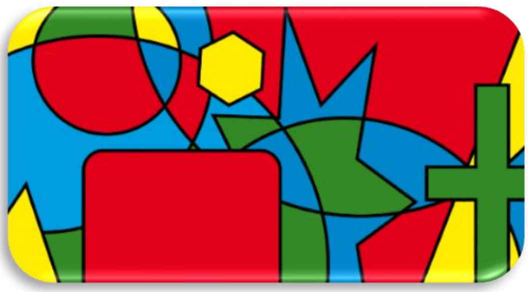
In 2016, a trio of mathematicians were able to prove the Boolean Pythagorean Triples proof by use of a computer programme. The proof that required 200-terabytes was unsolvable to mathematicians for decades. The assertion was that all positive integers could be coloured either red or blue with the property that no set of three integers a, b and c that satisfy $a^2 + b^2 = c^2$ (Pythagoras theorem) are all the same colour [12]. For example, for the Pythagorean triple 3, 4 and 5, if 3 and 5 were coloured blue, 4 would have to be red. This proof is so complex that it would take 10 billion years for 1 mathematician or 100 years for 100,000,000 mathematicians to solve. It took the computer just 2 days to solve it. The diagram above shows one solution for numbers 1-7,824 (white square can either be red, or blue). However, for numbers 1-7,825 there is no solution.



In 1637, Fermat stated, in its famous Fermat's last theorem, that for the more general equation $a^n + b^n = c^n$ had no solutions in positive integers (for any integer value of n greater than 2). It took more than 350 years until the first successful proof by Andrew Wiles in 1994.

Four-Colour Theorem

The conjecture first introduced in 1850's by Francis Guthrie stated that 'four colours suffice to colour any planar map so that no adjacent countries are of the same colour' [5]. The proof, which came over a century later, holds great importance in the history of mathematics, as it was the first major mathematical theorem to be proved with the help of computers. It was the work of Kenneth Appel and Wolfgang Haken in 1976 that solved it. Their approach was to set up counter examples and use this against the conjecture. They undertook proof by exhaustion by 'constructing an unavoidable set of configurations, making cases finite' [5]. It was known that if no counter example could exist then the theorem was proved. Due to the large dependency placed on the computer, it is almost impossible to be checked by humans, creating a question of whether their work could be 'considered 'proved' in the usual sense' [6]. As it was the first-time computers and AI had dependence to solve a proof, mathematicians had concerns that there may be a 'hidden flaw' in the technology used that may 'undermine the overall logic of the proof' [7]. It was a big step for AI when it helped solve the famous four-colour theorem. AI and computer technology had helped solve a proof that baffled mathematicians for over a century.



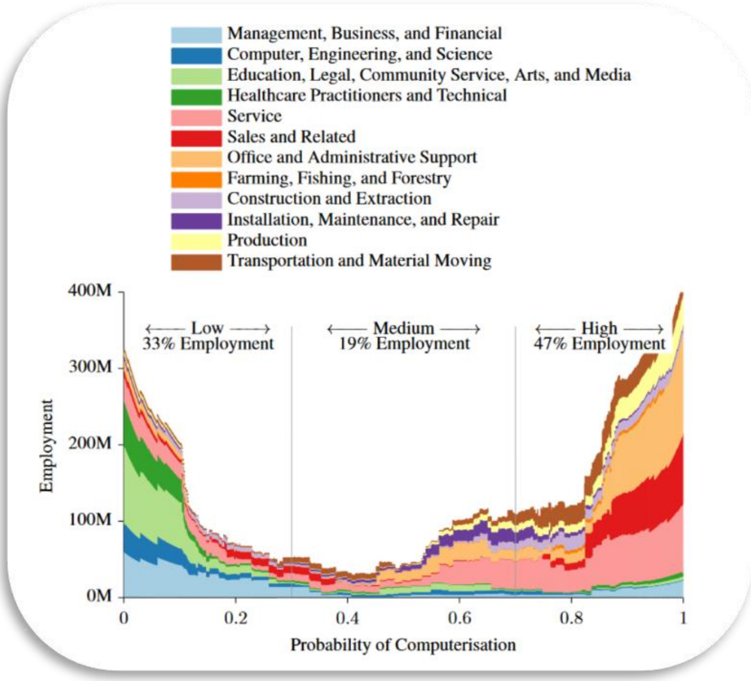
Kepler Conjecture

A conjecture that asks, 'how can three-dimensional spheres be packed as tightly as possible, so they take up the least amount of space?' [8] has puzzled mathematicians for centuries. The conjecture was first proposed by Johannes Kepler in 1611 and had remained unsolvable for almost 400 years. It was only in 1998 that Thomas Hales and his assistant Sam Ferguson proved the conjecture by use of 'a variety of computerized math techniques' [1]. The results Hales and Ferguson produced took up 3 gigabytes of storage which 'consisted of 300 written pages and an estimated 50,000 lines of computer code'. What Hales did was to 'reduce the infinite possibilities to a list of a few thousand densest graphs, setting up a proof-by-exhaustion' [9]. His proof was then checked by 12 mathematicians over many years until they were satisfied that his proof was 99% accurate. This was another breakthrough in the field of computerised mathematics. By correctly solving this proof with the use of a computer, Hales proved that 'computers can be used to reason in mathematics' [8], according to the philosophy professor, Jeremy Avigad.

From the examples above, it is clear that the development and implementation of AI has helped solve problems and proofs that mathematicians have spent their whole lives trying to solve. Hales strongly believes that the use of AI in formal proofs is what will help develop mathematics in the future. If AI can be used to solve large complex proofs that would have required lifetimes of work to solve by hand, 'then theoretically it would be possible to verify almost any mathematical result' [8]. Therefore, what then is stopping AI from taking over and making mathematicians obsolete? If automated reasoning is achieved, will this happen?

Frey and Osborne Graph of Automatable Jobs

Are mathematicians 'threatened with obsolescence due to rapidly advancing technology?' [10]. The graph produced by Carl Benedikt Frey and Michael Osborne in 2013 explores 'how susceptible jobs are to computerisation' [11]. According to this study, around 47% of total US employment is at risk. They predict that due to advances in AI and machine learning, the 'demand for labour input in tasks that can be routinised by means of pattern recognition' will dramatically decrease, while demand for tasks that are not 'susceptible to computerisation' will not. They describe that the probability axis can be a timeline of events, where jobs with a high probability of being automated will be substituted soon. They argue that engineering and science occupations are at low risk 'due to the high degree of creative intelligence they require'. The research displays 'strong complementarities between computers and labour in creative science and engineering occupations'.

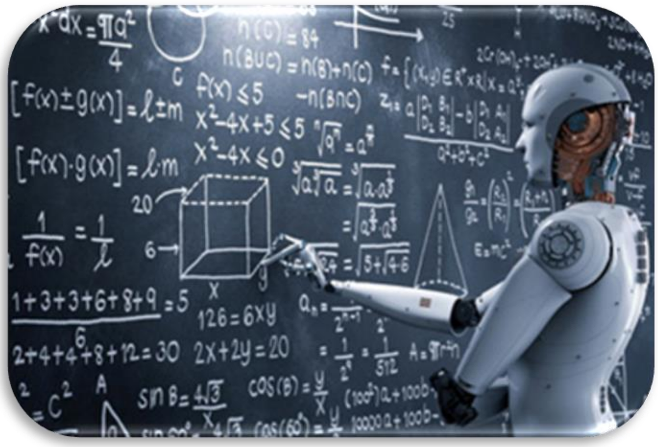


They believe that the task of a mathematician is to 'conduct research to extend mathematical knowledge in traditional areas, such as algebra, geometry, probability and logic', all of which require intuition and brave leaps that cannot result from artificial intuition. While Frey and Osborne perceive mathematics to be a profession with low probability of computerisation, they do not rule out the chance that it could happen in the future and technology matures and iterates.

Conclusion

The future of mathematics and mathematicians is difficult to predict, even with AI. AI is used right now to crunch through mundane, complex stages in proofs. It is already carrying out calculations and simplifications that a human could not do, especially without errors. Theorem packages such as Microsoft Z3 and algorithms such as the Wilf-Zeilberger are proving, simplifying, and calculating mathematical questions at a speed and accuracy humans could never achieve. The real question is whether 'synthetic intuition' can ever be programmed [1]. As discussed before, 'creativity is a fundamentally human activity' [2] - can we automate this? Proving a conjecture is one thing but having the intuition and creativity to discover and design a conjecture is another. As Galileo once wrote, 'all truths are easy to understand once they are discovered; the point is to discover them'. Only once computers and AI are able to imagine and develop their own conjectures, will machine automated intuition begin. Automated understanding 'suggests the possibility of automating reasoning itself' [1].

AI is leading the way in theorem proving and is paving its way to formal proof. According to Frey and Osborne, this will not result in making mathematicians obsolete for a long time, but there is still a chance that it might, eventually. Mathematicians are embracing this new way of working and the field of mathematics in continually expanding with the help of AI.



Should mathematicians worry about becoming obsolete? There is no obvious answer. If one looks at other sectors, AI has dramatically taken over. Jobs such as composing and translating have been replaced by a more efficient and accurate form of AI as there is no need for intuition. Until artificial intuition is automated, and we arrive to an era of real AGI (Artificial General Intelligence), I believe mathematicians should not worry. However, at that point, a more general question will be even more relevant: should humanity worry about AI and AGI?

Technology can be harnessed by individuals to their own advantage and therefor disadvantage of others, and if in the right or subsequently wrong hands, AI has huge potential for evil. Mathematicians must consider how to share knowledge, how to protect human kind and what ethical and moral guidelines should be established. With future implications in mind, will mathematicians, scientists and states come together to create a code of future conduct?

Is AI a friend or foe? No one knows. The technology being used to create theorem provers and solving proofs can be used to control airplanes, nuclear warheads and stock exchanges.

The biggest issue is, what if there is a bug? What will happen when artificial reasoning and intuition is created? Will AGI take over? Will AGI help life flourish like never before, or will machines outsmart us at everything [12], and even, perhaps, replace us altogether? Now more than ever is the time to consider the ethical, moral and practical dilemmas raised by this developing science. This conundrum is the one our mathematicians should focus on, to make sure the former, rather than the latter, is the future we are preparing for ourselves and our posterity.

Word Count: 2468

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